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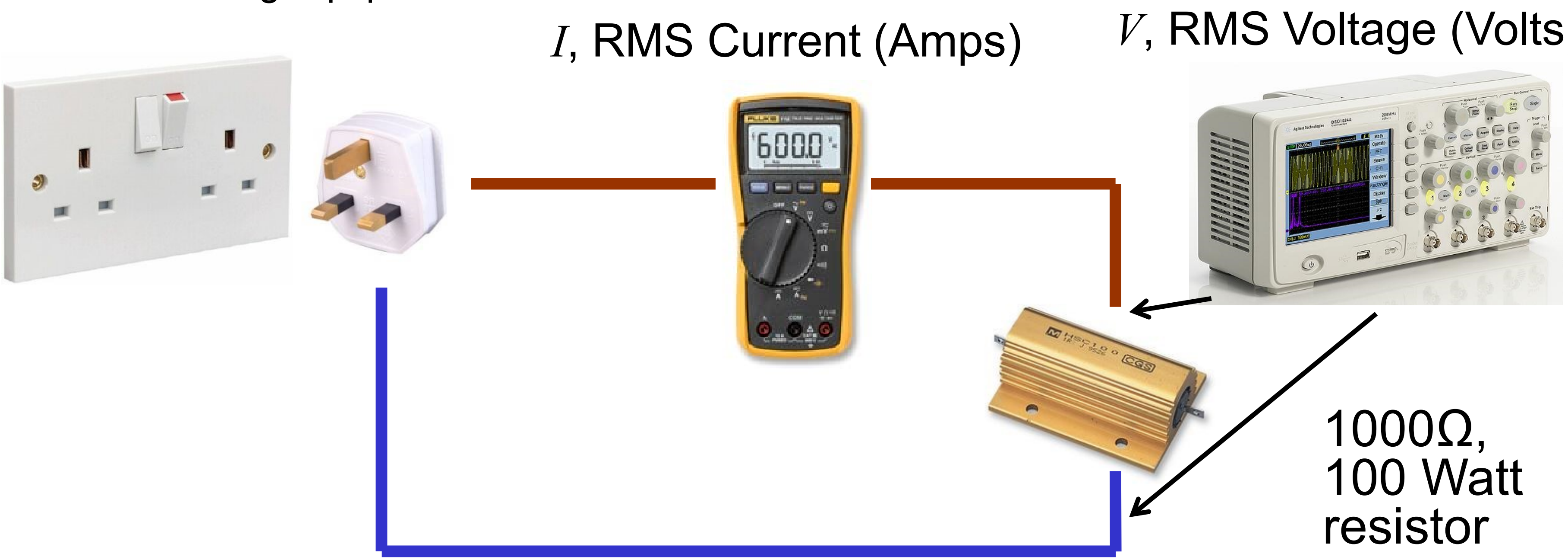
How accurate is a measurement?

Why should you care?

Dr. Andrew Roscoe

Does $V=IR$?

You are asked to confirm the hypothesis that $V=IR$.
The following equipment is used:



You measure RMS AC current on the digital multimeter (DMM), using the 1 Amp scale, as 0.213A

You measure RMS AC voltage on the oscilloscope as 239.2V

What do you conclude?

You would expect $V=IR=0.213\text{A} \times 1000\Omega = 213\text{V}$.

239.2V is ≈12% higher than you would expect.

What do you conclude?

Does $V=IR$?

You would expect V=IR=0.213A x 1000Ω = 213V.

239.2V is ≈12% higher than you would expect.

What do you conclude?

- We must have done something wrong
- The equipment is broken
- $V \neq IR$
- This is a stupid experiment

The hypothesis that $V=IR$ is still valid, given the uncertainties of the experimental setup.

Historical measurement standards in markets : weight

Historical measurement standards in markets : liquid volume

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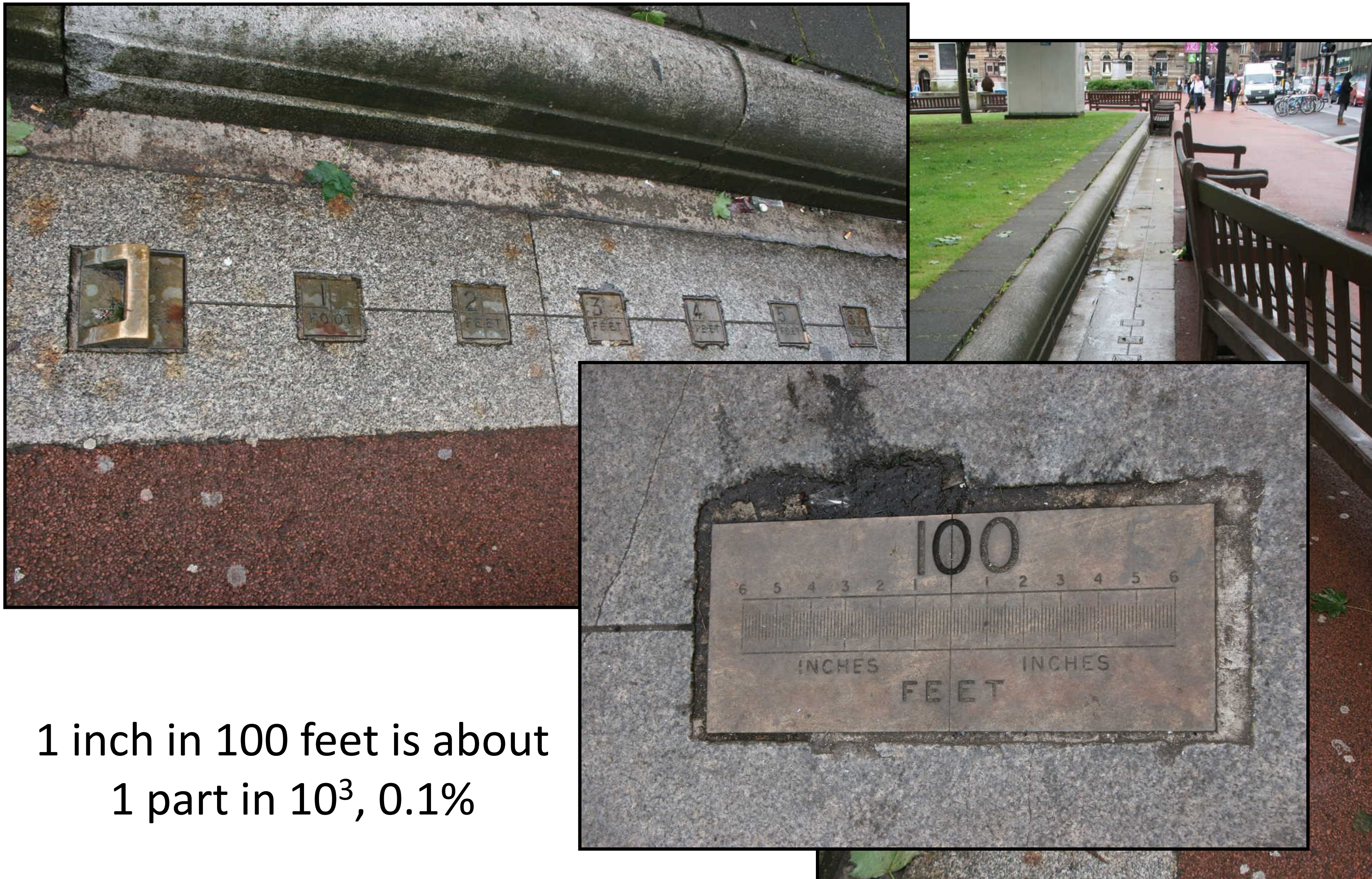
Historical measurement standards in markets : length

Historical measurement standards in markets : inches and feet

Historical measurement standards in markets : 100 feet



1 inch in 100 feet is about
1 part in 10^3, 0.1%

Historical measurement standards in markets : 1 chain

The 7 base units

- 1875 - The *Metre Convention* treaty
- 1954-1960 – *Le Système international d'unités* (SI)

Quantity	Dimension	Unit name	Unit symbol
l	L	Metre	m
m	M	Kilogram	kg
t	T	Second	s
I	I	Ampere	A
T	Θ	Kelvin	K
I_v	J	Candela	cd
n	N	Mole	mol

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The 7 base units

SI Base Units

A	cd	K	kg	m	mol	s
ampere	candela	kelvin	kilogram	metre	mole	second

- The standards for the base units (and other derived units) are defined and maintained by the national measurement institutions of many countries; for example …

NPL
National Physical Laboratory

LNE
Le progrès, une passion à partager

VSL
Dutch Metrology Institute

NIST
National Institute of Standards and Technology
U.S. Department of Commerce

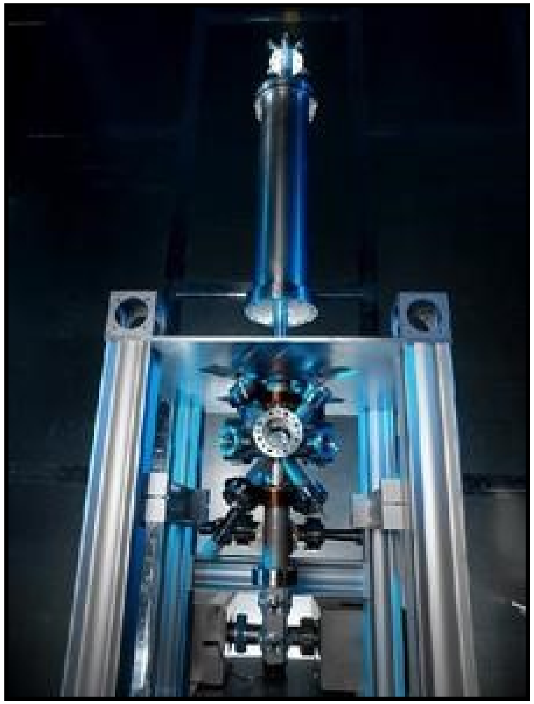
METAS
Federal Office of Metrology

Common Derived units

Unit name	Unit symbol	Example Derivation	Example in Common Units	SI base units
Newton	N	*F=ma*		kg m s^{-2}
Joule	J	*Energy = Force . Distance*	NM	kg m^2 s^{-2}
Watt	W		J/s	kg m^2 s^{-3}
coulomb	C	*1 Amp for 1 Second*	I	A s
Volt	V	*P=VI*	W/A	kg m^2 s^{-3} A^{-1}
Ohm	Ω	*V=IR*	V/A	kg m^2 s^{-3} A^{-2}

The modern time and frequency standard

- 1 second : the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom, at rest, and at a temperature of absolute zero.



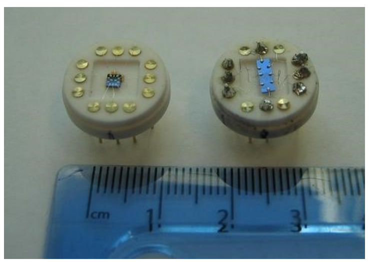
- Using a “Caesium fountain”, accurate to 1 second in 60 million years (1 part in $2x10^{15}$)

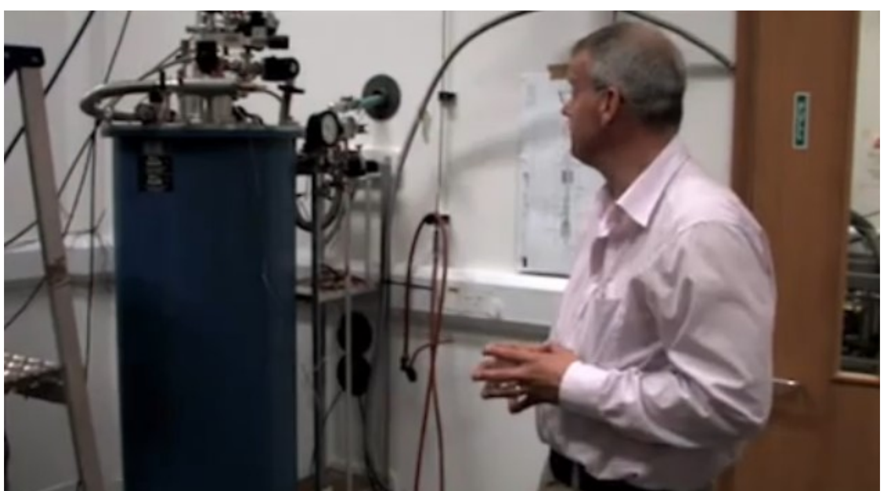
Amps and Volts

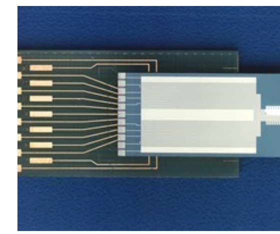
- 1 Amp : the ampere is that it is the constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between those conductors a force equal to 2 x 10^{-7} newtons per metre of length.
- This is impractical, so instead, standard current is measured using resistance and voltage

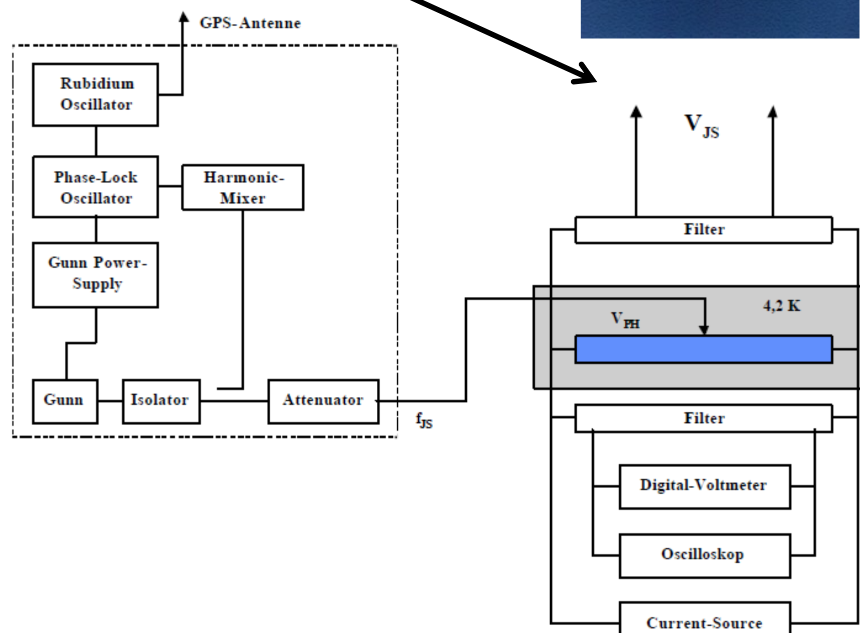
~0.05 ppm (1 part in 2x10^7)

$$I = \frac{V}{R}$$





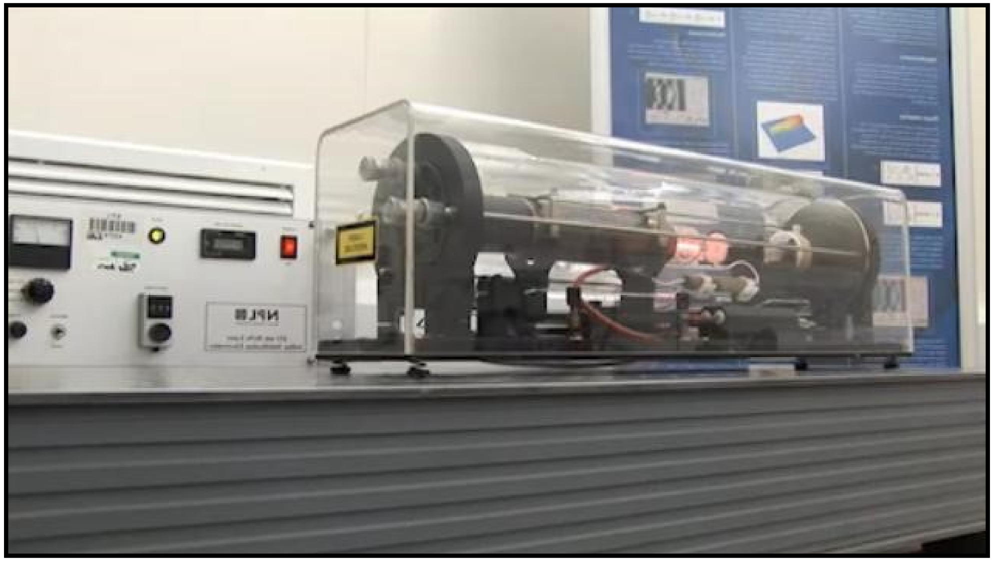




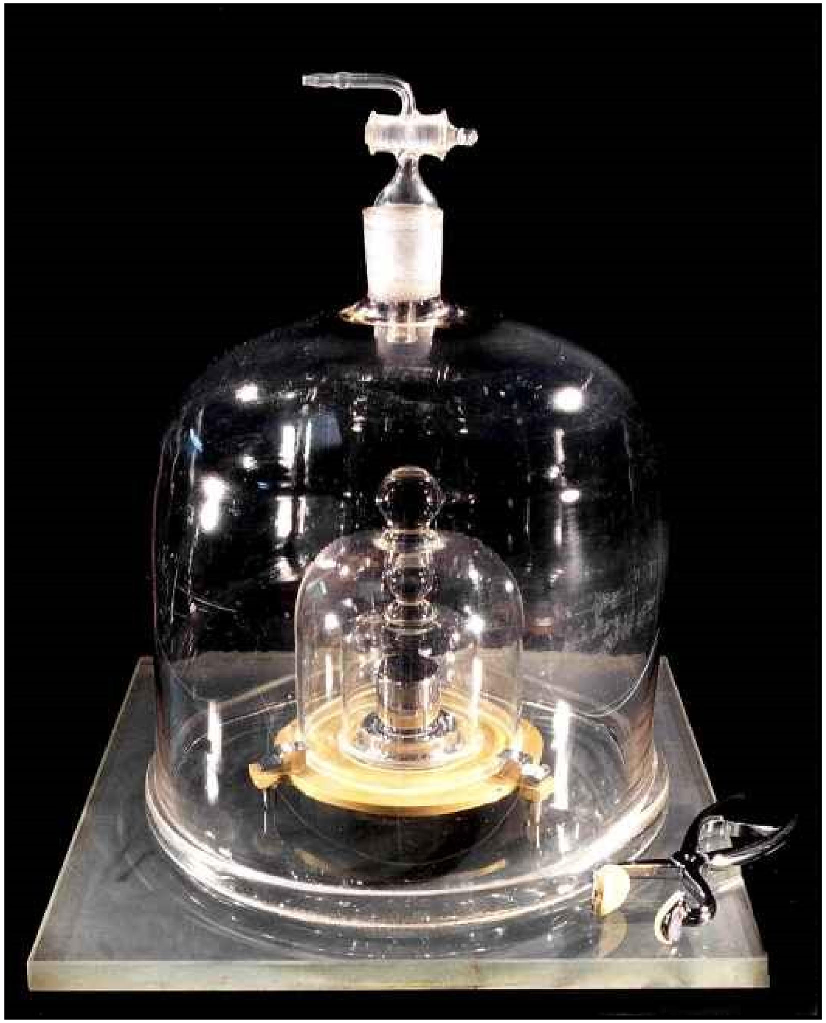
accurate to 1 part in 10^8 using Josephson junctions

Recent and current modern length standards

- 1 metre : the length of the path travelled by light in vacuum during a time interval of 1/ 299 792 458 of a second



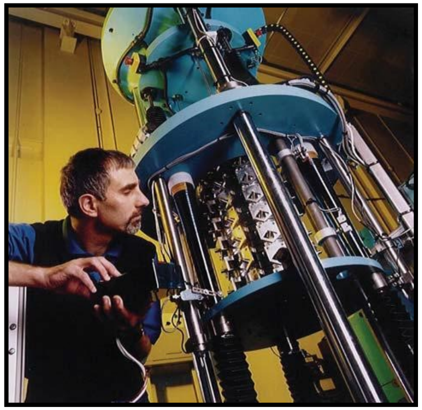
The “modern” kilogram standard



- It was made in 1879!
- The mass (kilogramme) standard is the ONLY standard which is still defined by a physical object.

Matters of scale:

How do you calibrate this weighbridge?



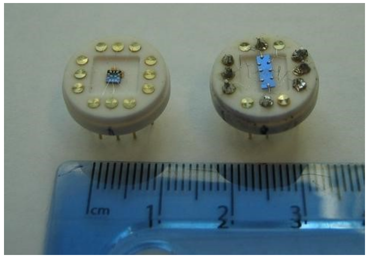
40 tonnes,
40000kg, ~0.4MN

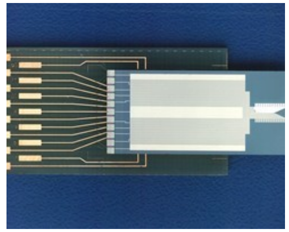
National Force Standard MachinesMeasurement range and uncertainties

Mode	Force Range	Force Uncertainty*	ISO 376 Class Capability	UKAS-Accredited Facility
Tension & Compression	1.5 N up to 25 N	±0.002 %	00	✓
	25 N up to 1.2 MN	±0.001 %	00	✓
	1.2 MN up to 5 MN	±0.05 %	1.0	✓
Compression Only	5 MN up to 10.8 MN	±0.10 % (incremental)	2.0	-
		±0.35 % (decremental)	N/A	-
	10.8 MN up to 30 MN	0.20 %	N/A	-

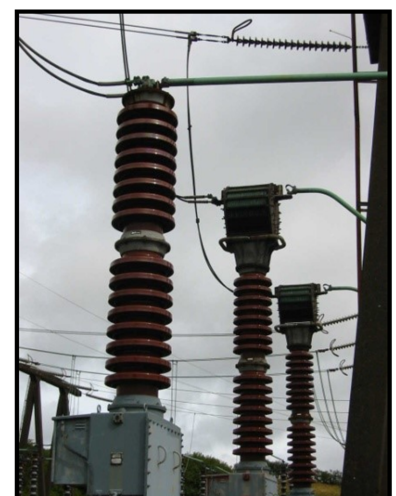
* This is an expanded uncertainty, giving a level of confidence of approximately 95%.

Matters of scale:





From such low-power voltage and current standards …

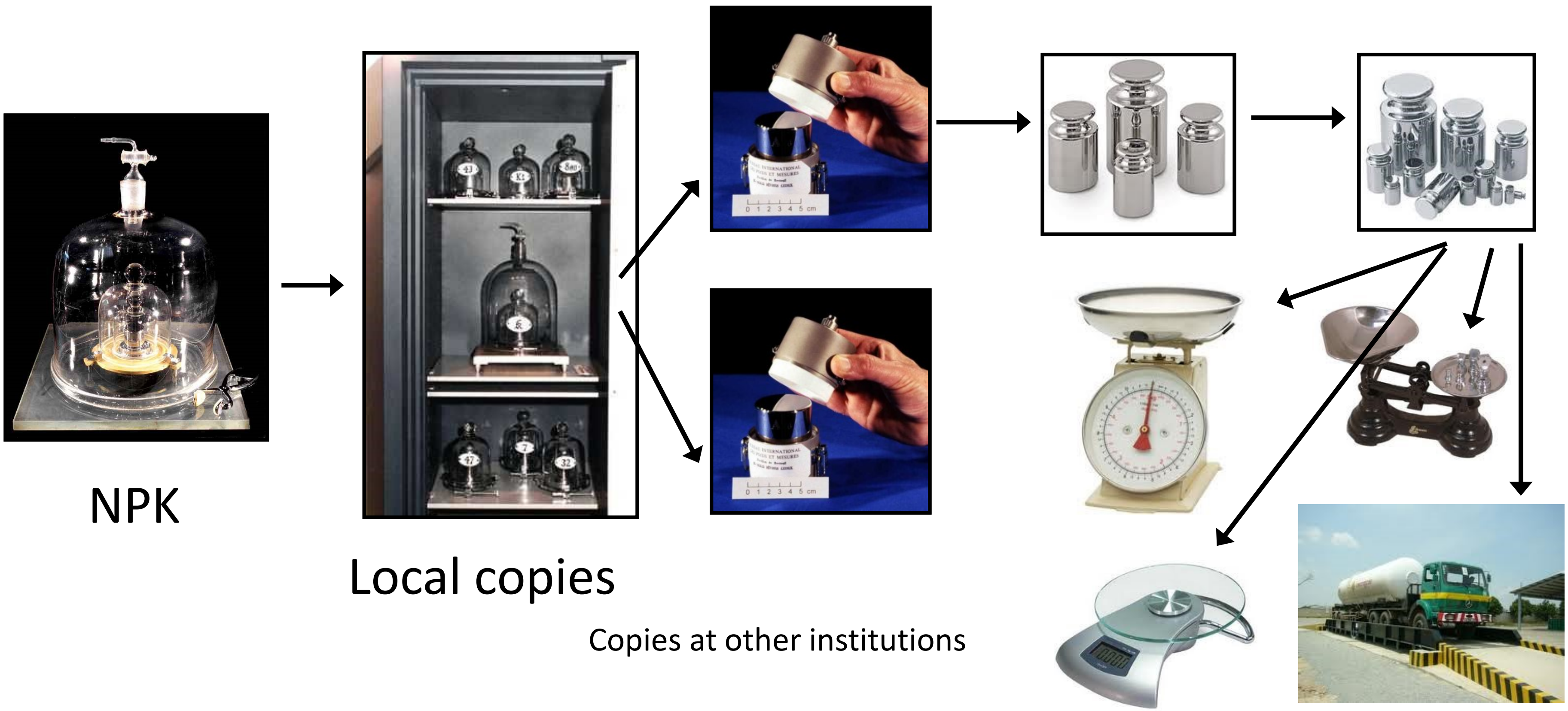


How do you calibrate AC voltages at 400kV - 1MV?

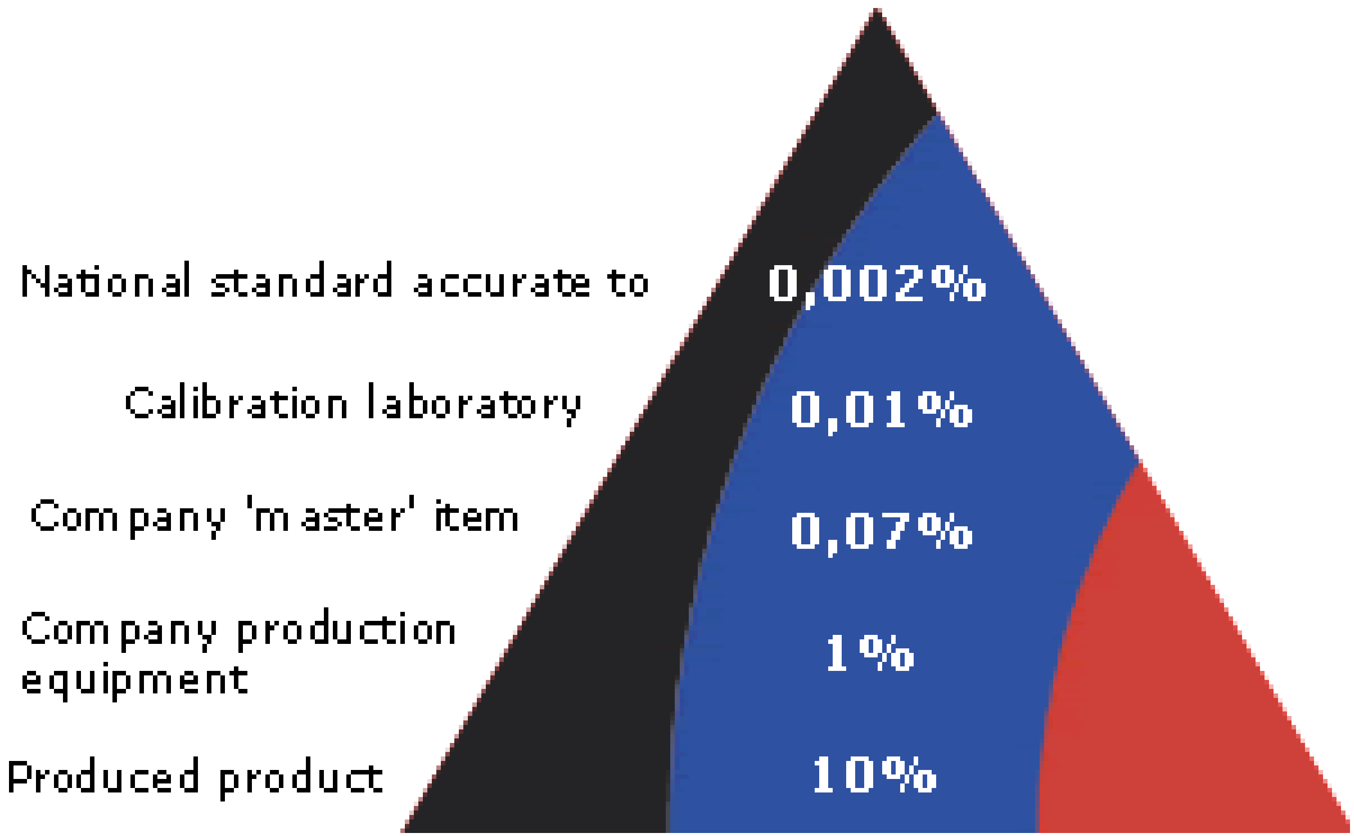
How do you calibrate AC currents up to 2000A?

Transferring standards

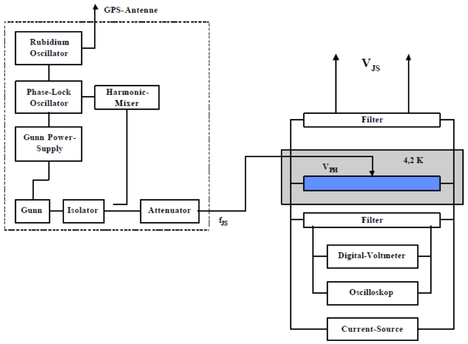
- The standard mass (the IPK, International Prototype Kilogram) is kept in France.
- In 1889, 40 copies were made. In the UK, NPL keeps copy #18.
- All the copies can be periodically checked against the IPK
- They are ALL measurably drifting against each other!

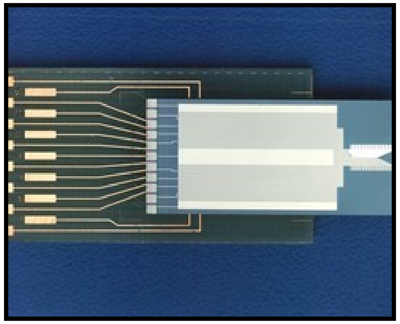


Transferring standards and traceability



Example of traceability : Voltage (1)





accurate to 1 part in 10^8 (0.01 ppm)
using Josephson junctions

Calibration accuracies offered by NPL

Voltage Level	Uncertainty (95 % confidence level)
1.0 V Electronic	0.14 ppm
1.018 V Electronic	0.14 ppm
1.018 V Standard Cell	0.09 ppm
10 V Electronic	0.02 ppm

0.02-0.2 ppm

0.01 ppm

1-2 ppm
£5000 each

Example of traceability : Voltage (2)

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Stability

Stability for a given period of time is defined as the output uncertainty minus the calibration uncertainty at the 99% Confidence Level. When the output voltage is characterized by a regression model, stability is given by the following equation:

$$\left| b\left(\frac{P}{365}\right) + 2.65 S_1 \sqrt{\left[\frac{S_{ra}}{S_1}\right]^2 + \left(\frac{1}{n}\right) + \left[\frac{\left(\bar{x} + P - x_1\right)^2}{\Sigma\left(X_j - \bar{x}\right)^2}\right]} \right|$$

where b = slope of regression in ppm/year

S_1 = standard deviation about the regression (SDEV)

S_{ra} = SDEV of data filtered with 7-day moving average filter (MAF)

P = Period of time under consideration in days

$\bar{x}$ = mean time for regession data

n = 180 period (typically 2 meassurements per day)

Xj = jth period

X_1 = time at beginning of data

Each data point for the computation of the regression parameters is the average voltage of 50 readings taken in a 50-second measurement period.

Stability for the 732B outputs at 23 ±1°C is specified as follows:

Output Voltage	Stability (± ppm)		
	30 Days	90 Days	1 Year
10V	0.3	0.8	2.0
1.018V	0.8	NA	NA

Noise at the Output Terminals

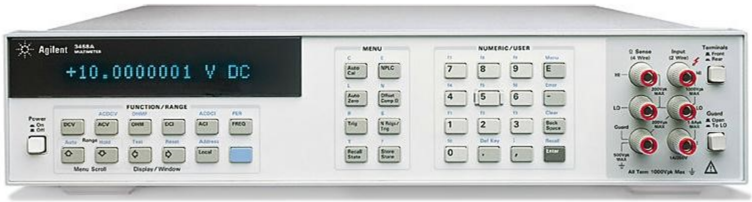
Output noise is specified for both day-to-day observations and for short-term observations. The former is given by the standard deviation of a 90-day regression model. The latter is in terms of its rms value in a bandwidth as follows:

Output Voltage	S_1 (± ppm)	S_{ra} (± ppm)	Noise (0.01 Hz to 10 Hz (± ppm rms)
10V	0.068	0.05	0.06
1.018V	0.1	NA	0.03

Output Current and Limits

Output Voltage	Output Current Limit	Output Impedance
10V	12 mA (Note)	≤ 1 mΩ
1.018V	20 pA	≤ 1 kΩ
Note: Limit output current to ≤ 0.1 mA to realize 72 hour battery operation.		

Example of traceability : Voltage (3)



8-ppm 1 year dcV accuracy, optional 4-ppm
0.05 ppm dcV transfer accuracy

HP/Agilent 3458A, £5600
~10ppm (0.001%)

Agilent DSO1024A
4%, £1500

Agilent 34410A, £850
.0030 % DC2, 0.06% ACV

Agilent 3401A, £300
0.02% DCV ,0.5% ACV

Fluke 115, £150
DCV 0.5% + 2
ACV 1% + 3 (45Hz -500Hz)
2% + 3 (500Hz-1kHz)

DCA 1% + 3
ACA 1.5% + 3
Resistance 0.9% + 1

Does $V=IR$?

You are asked to confirm the hypothesis that $V=IR$.
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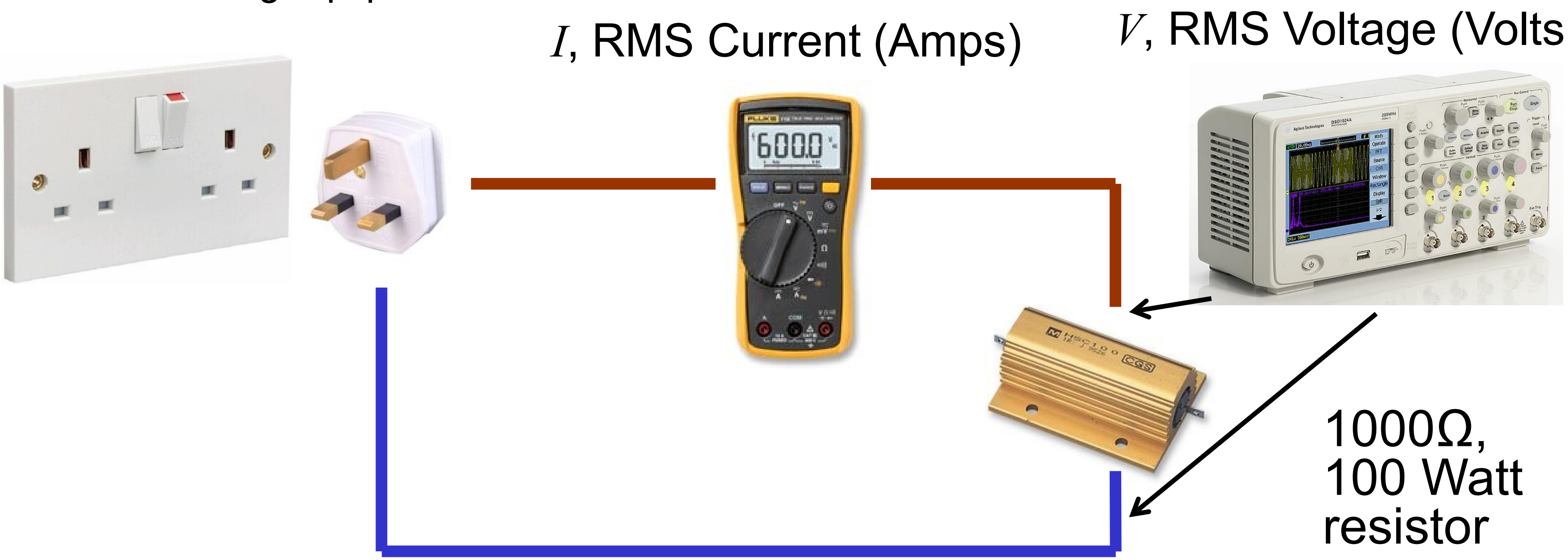

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What do you conclude?

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239.2V is ≈12% higher than you would expect.

What do you conclude?

Does $V=IR$?

You would expect V=IR=0.213A x 1000Ω = 213V.

239.2V is ≈12% higher than you would expect.

What do you conclude?

- The actual current could be higher, by 3 digits and 1.5% than measured (0.213A), i.e. 0.219A. We would expect a voltage of V=IR=0.219A x 1000Ω = 219V.
- The actual voltage could be lower than measured (239.2V)by 4%, i.e. 229.6V
- This would still be higher than expected by 5%

Fluke 115, £150

ACA 1.5% + 3

Agilent DSO1024A

4%, £1500

It still looks like $V \neq IR$

Does $V=IR$?

The measured voltage still appears to be ≈5% higher than you would expect, given the measurement uncertainties.

What do you conclude?

- But the resistor is not ACTUALLY 1000Ω
- It has 5% accuracy at room temperature
- PLUS, a temperature coefficient of 30ppm/°C, and it is at 125°C due to the power dissipation.
- Its REAL resistance could be $1000*(1.05+30*10^{-6}*100)=1053\Omega$, 5.3% higher that 1000Ω

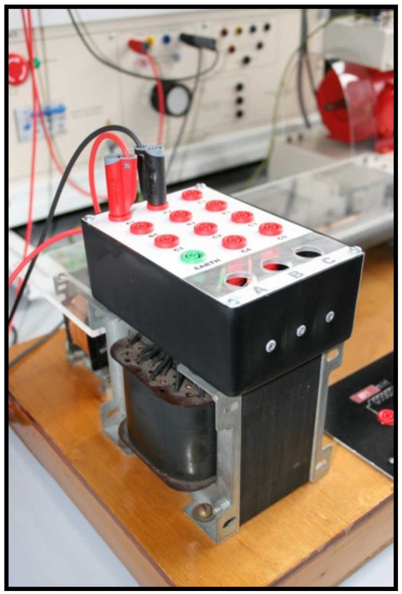
1000Ω, 100 Watt resistor

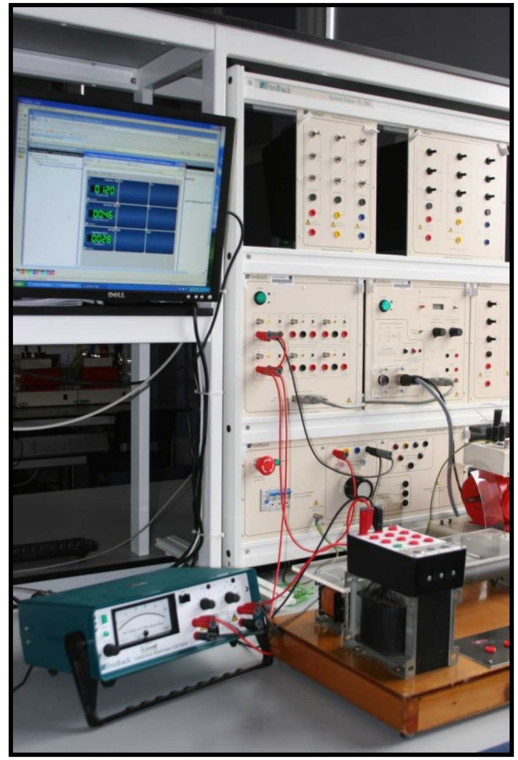
V could be equal to IR

Approximate overall errors

- AC current : 3%
- AC voltage : 4%
- Resistance value : 5.3%
- Total potential error : **12.3%**

Poor measurement algorithms (1)





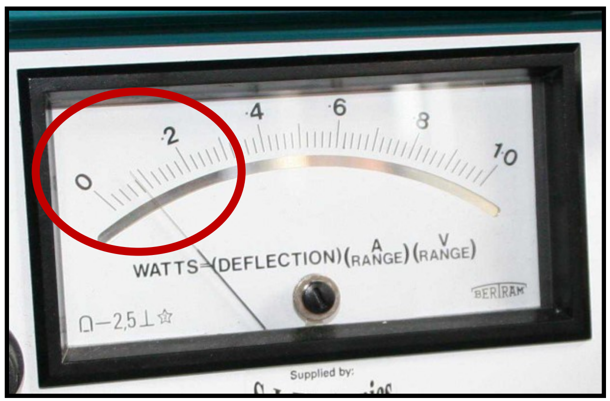
31 Watts ! ?

Measure the power consumption of an open-circuit transformer, using the newest laboratory equipment

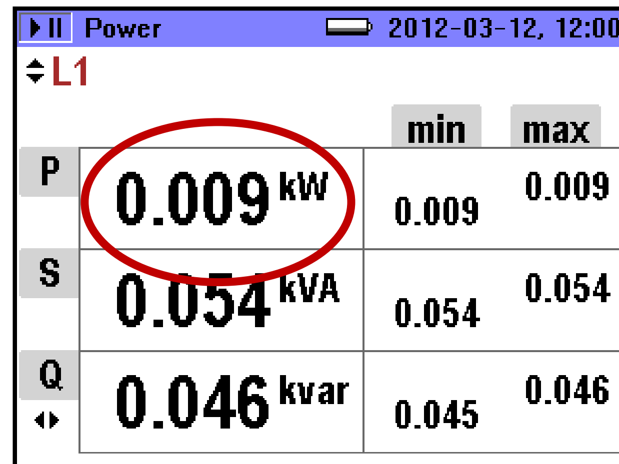
Poor measurement algorithms (2)

Error ~200-300% !!

31 Watts

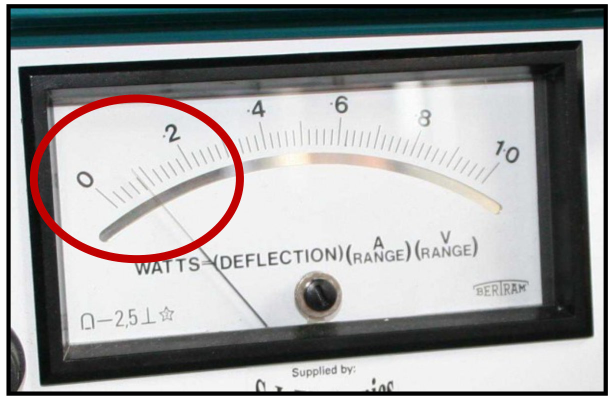


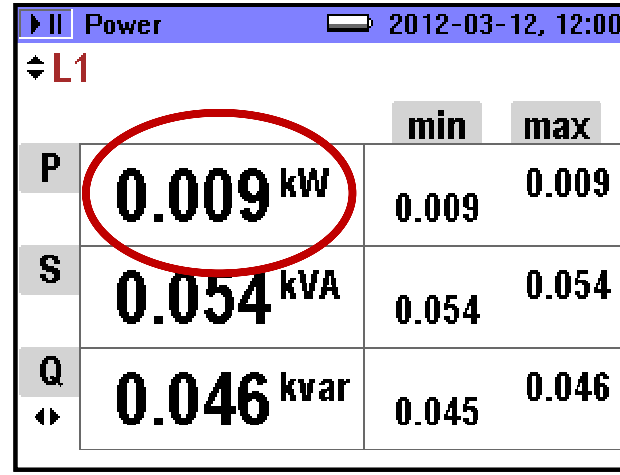
8 Watts

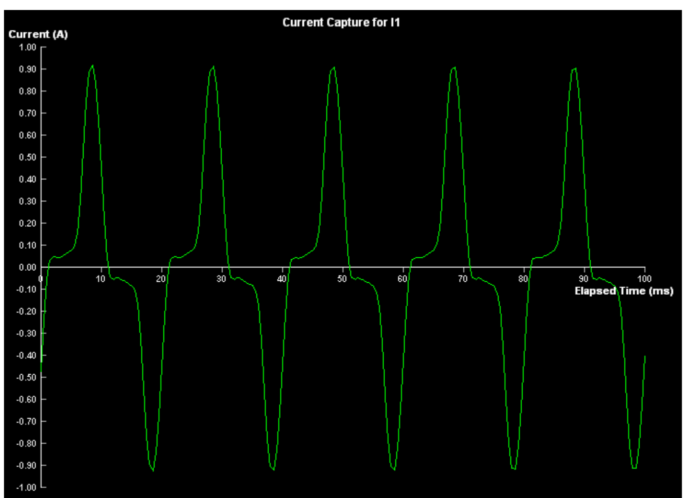


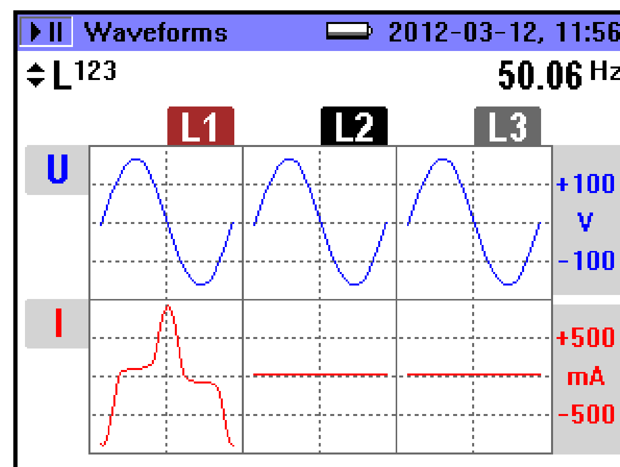
9 Watts

Poor measurement algorithms (3)

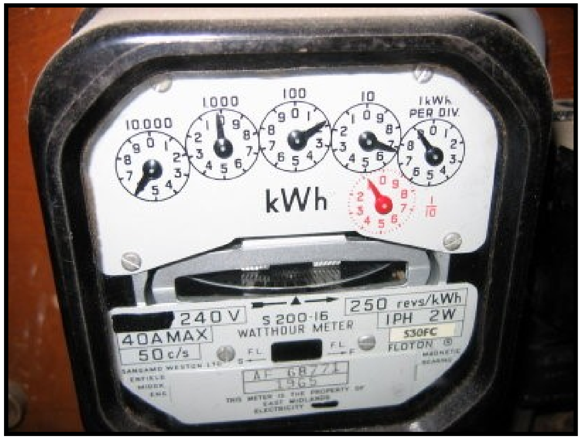


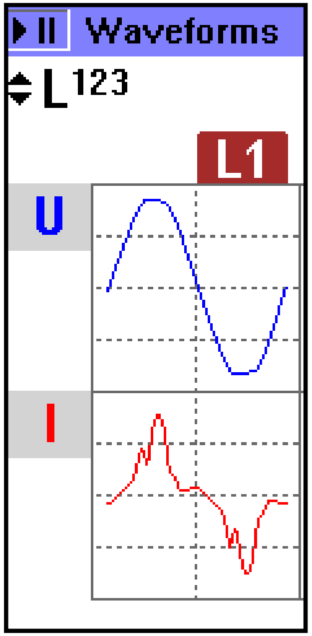






How accurate is your electricity meter?





+2.5% and -3.5% for UK nationally approved meters. *But what about harmonics?!*

Example of traceability of RF power measurement

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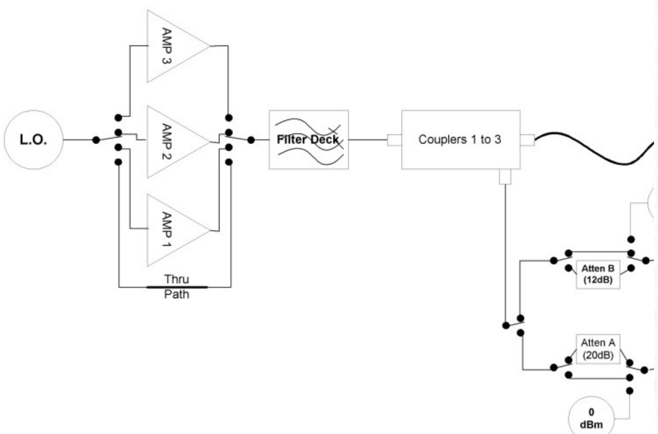

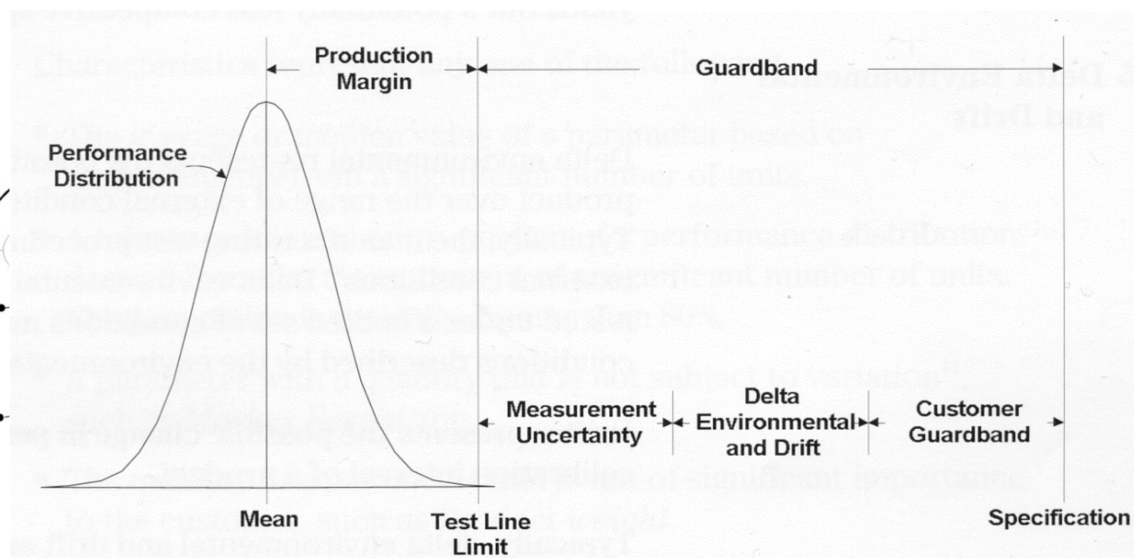

Resistance

Resistance calibrations

Low power class S resistors

NPL provides calibration services for resistance standards ranging from 0.1 mΩ to 1 GΩ. Measurement capability exists for low power dc measurements, high power dc measurements of current shunts and ac impedance measurements.

All measurements of resistance at NPL are referred to the quantised Hall effect using the conventional value of the von-Klitzing constant R_{K-90} = 25 812.807 Ω exactly. The uncertainty of the value of R_{K-90} in the SI system is not included in the uncertainty assessments.

Two 100 Ω room temperature resistors are measured in terms of the quantised Hall effect on a regular basis using a resistance bridge based on a cryogenic current comparator. These resistors are used as day-to-day working standards, and all other resistance standards at NPL are related to these resistors using a variety of measurement techniques.

Low power resistors

Resistors designed for low power dissipation are normally measured at a power of 1 mW or less. Standards are measured in either an oil bath or an air bath at measurement temperatures of 20 °C or 23 °C (other temperatures are available on request). Resistors at decade values and at 25 Ω are calibrated as standard with the uncertainties given in the table below. Other values can be calibrated on request.

High power resistors

Calibration of high power resistors is offered at powers up to 100 W and currents of up to 100 A.

Temperature coefficient of resistance

Resistors submitted for temperature coefficient determination are measured at several temperatures in the range from 17 °C to 25 °C and the measurements are fitted to a second order polynomial.

AC impedance of resistors

Resistors in the range 1 Ω to 10 kΩ can be measured at frequencies from 40 Hz to 20 kHz. The uncertainty quoted varies with resistor value and frequency. The best uncertainty currently available is 0.5 ppm for the real part of the impedance and 10 ns for the time constant.

Nominal Value	Uncertainty (95 % confidence level)
100 μΩ	2.5 ppm
1 mΩ	0.85 ppm
10 mΩ	0.8 ppm
100 mΩ	0.18 ppm
1 Ω	0.06 ppm
10 Ω	0.05 ppm
25 Ω	0.05 ppm
100 Ω	0.05 ppm
1 kΩ	0.05 ppm
10 kΩ	0.06 ppm
100 kΩ	0.08 ppm
1 MΩ	0.12 ppm
10 MΩ	0.2 ppm
100 MΩ	0.4 ppm
1 GΩ	1.6 ppm

END